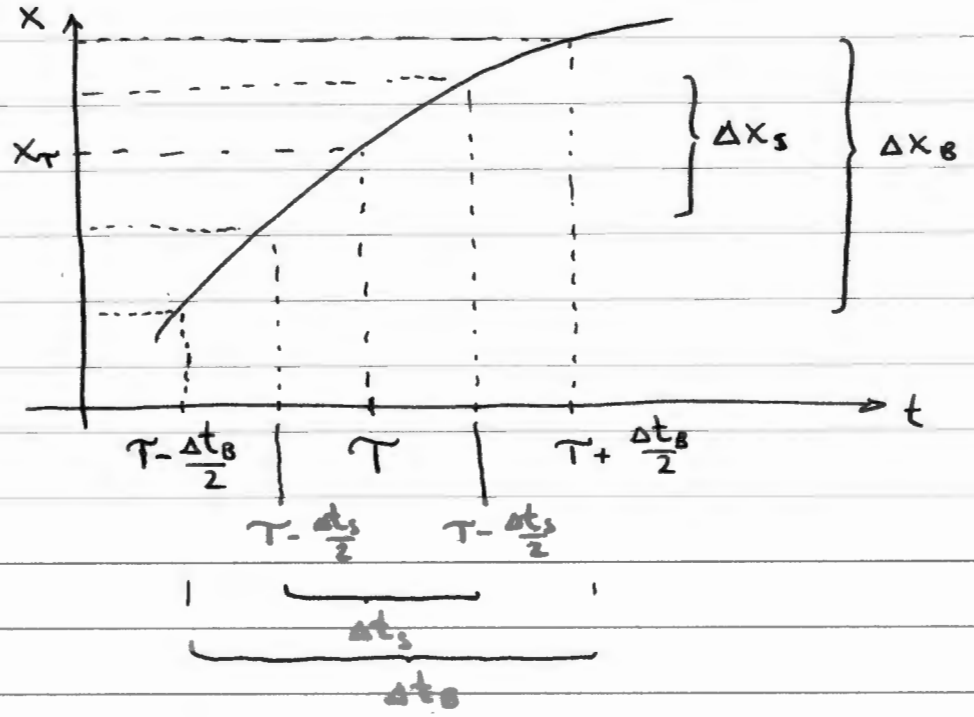
Velocity & Finding Derivatives



$$\overline{V}_{Big} = \frac{\Delta x_B}{\Delta t_B} \quad , \quad \overline{V}_{small} = \frac{\Delta x_s}{\Delta t_s}$$

we continue to make the Δt be smaller and smaller time intervals

eventually we get to the point where we have the line that is tangent to the curve at the point T, the slope of the curve

this is the velocity at T, $V(T)$

we can slide T along to map out the velocity at any time

$$V(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta t \to 0} \left[\frac{x(t+\frac{\Delta t}{2}) - x(t-\frac{\Delta t}{2})}{(t+\frac{\Delta t}{2}) - (t-\frac{\Delta t}{2})} \right]$$

$$= \frac{dx}{dt}$$

so to get our derivative we find the values of x at $t+\Delta t/2$ & at $t-\Delta t/2$ and find the difference, and divide by the difference in time $(t+\Delta t/2)$ minus $(t-\Delta t/2)$

I will show the first special case we will be using, where $x(t)$ is a constant

$$x(t) = x_0$$

then $$v(t) = \frac{dx}{dt} = \lim_{\Delta t \to 0}\left[\frac{\Delta x}{\Delta t}\right]$$

$$= \lim_{\Delta t \to 0}\left[\frac{x(t+\Delta t/2) - x(t-\Delta t/2)}{(t+\Delta t/2) - (t-\Delta t/2)}\right]$$

$$= \lim_{\Delta t \to 0}\left[\frac{x(t+\Delta t/2) - x(t-\Delta t/2)}{\Delta t}\right]$$

now use our function $x(t) = x_0$ & then take the limit

$$v(t) = \lim_{\Delta t \to 0}\left[\frac{x_0 - x_0}{\Delta t}\right]$$

$$= \lim_{\Delta t \to 0}\left[\frac{0}{\Delta t}\right] = \lim_{\Delta t \to 0}(0)$$

$$= 0$$

1. The first problem for homework is to follow the same procedure for the function

$$x(t) = \alpha t$$

That is, substitute into

$$\frac{dx}{dt} = \lim_{\Delta t \to 0} \left[\frac{x(t + \Delta t/2) - x(t - \Delta t/2)}{(t + \Delta t/2) - (t - \Delta t/2)} \right]$$

and then do any possible algebra before you finally let $\Delta t = 0$

2. The second problem for homework is to follow the same procedure for the function

$$x(t) = \beta t^2$$